CESSA WP 2016-01

Decomposition of gross exports into value-added: an alternative accounting system

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October 2016

Center for Economic and Social Studies in Asia (CESSA) Working Paper

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Abstract

We demonstrate that we can trace decomposition processes of gross exports into value-added or factor contents across countries in a straightforward manner at both the non-bilateral and bilateral levels, if we redefine value-added exports, based on Trefler and Zhu (2010). For a bilateral trade system, we witness double counts as well as additive counts. We also provide a numerical example for a three-country case (China, the USA, and the rest of the world).

JEL E01, E16, F14, F23, L14

Keywords: Value-added exports, gross exports, decomposition, bilateral trade

1. Introduction

In light of the growing intermediate goods trade, major international organizations including the World Trade Organization (WTO) have emphasized the importance of a new concept of trade in value-added (TiVA) in place of traditional gross trade in order to capture global value chains through foreign trades across countries. In this paper we present an alternative accounting system of gross exports at both the non-bilateral level and the bilateral level[1] in place of that of Koopman et al. (2014). First, we

[1] The non-bilateral level focuses on a country's exports from, and imports to, the world, while the bilateral level considers exports and imports between two countries (if one is the origin/source country, another is the destination country). For a two-country case, we do not have to distinguish these two levels.

re-define non-bilateral and bilateral value-added exports by using elements of the value-added ratios, international Leontief inverse, and gross exports/imports based on Trefler and Zhu (2010). Second, we prove that our definition of bilateral value-added exports is mathematically equivalent to Johnson and Noguera (2012)'s value-added exports defined by using elements of the value-added ratios, international Leontief inverse, and destination country's final demand. Third, we demonstrate that we can trace gross exports through the decomposition of gross exports into value-added, double and additive counts in a straightforward manner if employing our representation of value-added exports in place of Johnson and Noguera's definition. For the non-bilateral trade system with more than three countries we can clearly witness two parts of return-home and no-return-home as double counts in the foreign content of gross exports. Particularly, we can clearly see that the latter, no-return-home as the double count, means return-home as the additive count at the bilateral level. We also provide a numerical example for a three-country case (China, the USA and the rest of the world), using aggregated World Input-Output Data, in order to enhance our alternative gross exports accounting system.

2. Model

Following Isard (1951), Trefler and Zhu (2010) and Johnson and Noguera (2012), we reproduce an inter-country multi-sector model in a general framework.

We assume that there are r, s =1, 2, …, R countries (areas or regions), each of which produces and inputs $r(i)$, $s(j)$ = 1, 2, .., n products. We further assume the classical Leontief open input–output model with fixed input coefficients and final demand for each country. In this model, each sector produces a single commodity without joint production. We regard the last country R as ROW (the rest of the world). We consider an international input–output system not in physical terms but in *value terms*. Table 1 shows the basic data structure of the system.

[Table 1 here]

We denote the following:

- $\boldsymbol{A}_{rs} = (a_{r(i)s(j)})$ $(n \times n)$: country r's export coefficient matrix to country s or country s's import coefficient matrix from country r if $r \neq s$, and country r's input coefficient matrix of domestically produced intermediate goods if $s = r$;
- $\boldsymbol{Y}_{rs} = [Y_{r(i)s}]$ $(n \times 1)$: country s's final demand vector for country r $(n \times 1)$ or country r's final goods export vector to country s if $r \neq s$;
- $\boldsymbol{Y}_r = [Y_{r(i)}]$ $(n \times 1)$: origin country r's final demand vector in an international input–output table (origin-wise final demand vector); $\boldsymbol{Y}_r = \Sigma_s \boldsymbol{Y}_{rs}$;
- $\widetilde{\boldsymbol{Y}}_r = [\widetilde{\boldsymbol{Y}}_{r(i)}]$ $(n \times 1)$: country r's final demand vector, including exports of intermediate goods, in each country's input–output system;
- $\boldsymbol{F}_s = [\boldsymbol{Y}_{rs}]$ $((n \times R) \times 1)$: destination country s's final demand vector for all countries (destination-wise final demand vector);
- $\boldsymbol{X}_{rs} = [X_{r(i)s}]$ $(n \times 1)$: origin country r's output vector induced by destination country s's final demand;
- $\boldsymbol{X}_r = [X_{r(i)}]$ $(n \times 1)$: origin country r's output vector (origin-wise output vector); $\boldsymbol{X}_r = \Sigma_s \boldsymbol{X}_{rs}$;
- $\boldsymbol{X} = [\boldsymbol{X}_r]$ $((n \times R) \times 1)$: an overall output vector;
- $\boldsymbol{X}_{*s} = [\boldsymbol{X}_{rs}]$ $((n \times R) \times 1)$: an overall output vector induced by destination country s's final demand (destination-wise output vector);

- $\boldsymbol{I}$: an ($n \times R$) dimensional identity matrix; and
- $\boldsymbol{I}_n$: an n-dimensional identity matrix.

We assume that non-negative matrixes $\boldsymbol{A}$ and $\boldsymbol{A}_{rr}$ are productive.

Denoting $\boldsymbol{X}^*$ as the equilibrium output vector, the global equilibrium (market clearing) condition for an Isard type of non-competitive inter-country multi-sector input–output table in value terms can be written as:

$$\boldsymbol{X}^* = \boldsymbol{A}\boldsymbol{X}^* + \boldsymbol{Y};\ \boldsymbol{X}^* = \boldsymbol{B}\boldsymbol{Y}, \text{where } \boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-1}, \tag{1}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \dots & \boldsymbol{A}_{1s} \dots & A_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{A}_{r1} & \boldsymbol{A}_{r1} & \dots & \boldsymbol{A}_{rs} \dots & \boldsymbol{A}_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{A}_{R1} & \boldsymbol{A}_{R2} & \dots & \boldsymbol{A}_{Rs} \dots & \boldsymbol{A}_{RR} \end{bmatrix},$$

$$\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \dots & \boldsymbol{B}_{1s} & \dots & \boldsymbol{B}_{1R} \\ \dots & \dots & \dots & \dots & & \dots \\ \boldsymbol{B}_{r1} & \boldsymbol{B}_{r1} & \dots & \boldsymbol{B}_{rs} & \dots & \boldsymbol{B}_{rR} \\ \dots & \dots & \dots & \dots & & \dots \\ \boldsymbol{B}_{R1} & \boldsymbol{B}_{R2} & \dots & \boldsymbol{B}_{Rs} & \dots & \boldsymbol{B}_{RR} \end{bmatrix}, \text{ and}$$

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_1 \\ \dots \\ \boldsymbol{Y}_r \\ \dots \\ \boldsymbol{Y}_R \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11} \\ \dots \\ \boldsymbol{Y}_{r1} \\ \dots \\ \boldsymbol{Y}_{R1} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{Y}_{1s} \\ \dots \\ \boldsymbol{Y}_{rs} \\ \dots \\ \boldsymbol{Y}_{Rs} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{Y}_{1R} \\ \dots \\ \boldsymbol{Y}_{rR} \\ \dots \\ \boldsymbol{Y}_{RR} \end{bmatrix} = \boldsymbol{F}_1 + \dots + \boldsymbol{F}_s + \dots + \boldsymbol{F}_R;$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \dots \\ \boldsymbol{X}_r \\ \dots \\ \boldsymbol{X}_R \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_{11} \\ \dots \\ \boldsymbol{X}_{r1} \\ \dots \\ \boldsymbol{X}_{R1} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{X}_{1s} \\ \dots \\ \boldsymbol{X}_{rs} \\ \dots \\ \boldsymbol{X}_{Rs} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{X}_{1R} \\ \dots \\ \boldsymbol{X}_{rR} \\ \dots \\ \boldsymbol{X}_{RR} \end{bmatrix} = \boldsymbol{X}_{*1} + \dots + \boldsymbol{X}_{*s} + \dots + \boldsymbol{X}_{*R}.$$

Overall output $\boldsymbol{X}^*_{*s}$ and origin country r's output $\boldsymbol{X}^*_{rs}$, induced by a fixed destination country s's final demand $\boldsymbol{F}_s$, are given by

$$\boldsymbol{X}_{*s}^{*} = \boldsymbol{A}\boldsymbol{X}_{*s}^{*} + \boldsymbol{F}_{s} = \boldsymbol{B}\boldsymbol{F}_{s}; \;\; \boldsymbol{X}_{*s}^{*} = (\; \boldsymbol{X}_{1s}^{*} \dots \boldsymbol{X}_{rs}^{*} \dots \;\boldsymbol{X}_{Rs}^{*})'; \boldsymbol{X}_{rs}^{*} = \Sigma_{k}\boldsymbol{A}_{rk}\boldsymbol{X}_{ks}^{*} + \boldsymbol{Y}_{rs}\,. \quad (2)$$

k =1,2, .., R also denotes country k, while (′) denotes the transpose of a matrix or a vector. This equation is essential for Johnson and Noguera's definition of value-added exports. By the given definitions of $\boldsymbol{F}_s$ and $\boldsymbol{Y}_{rs}$, we have $\boldsymbol{X}^* = \Sigma_s \boldsymbol{X}_{*s}^*$; ; $X_{rs}^* = \Sigma_i X_{r(i)s}^*$. Note that the origin-wise output $\boldsymbol{X}_r$ or $\boldsymbol{X}_s$ is different from the respective destination-wise output $\boldsymbol{X}_{*r}$ or $\boldsymbol{X}_{*s}$.

In view of intermediate gross exports (imports), $\underline{\boldsymbol{E}}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}^{*}{}_{s}$ in Table 1**,** country r's gross exports to country s, $\boldsymbol{E}_{rs}$ and its total gross exports, $\boldsymbol{E}_r$ are given by

$$\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}^{*}{}_{s} + \boldsymbol{Y}_{rs}\,(s \neq r); \; \boldsymbol{E}_{r} = \Sigma_{s \neq r}\boldsymbol{E}_{rs} \;\;, \text{ where } \boldsymbol{X}_{s}^{*} = \Sigma_{k}\boldsymbol{X}_{sk}^{*}. \quad (3)$$

Hence, the local equilibrium (market clearing) condition that each country must satisfy is given by

$$\boldsymbol{X}_{r}^{*} = (\boldsymbol{I}_{n} - \boldsymbol{A}_{\boldsymbol{rr}})^{-\boldsymbol{1}}\widetilde{\boldsymbol{Y}}_{r}, = \; (\boldsymbol{I}_{n} - \boldsymbol{A}_{\boldsymbol{rr}})^{-\boldsymbol{1}}(\boldsymbol{E}_{r} + \boldsymbol{Y}_{rr}). \quad (4)$$

This can also be written as $\boldsymbol{X}_r^* = \boldsymbol{B}^r\widetilde{\boldsymbol{Y}}_r$, where $\boldsymbol{B}^r = (\boldsymbol{I}_n - \boldsymbol{A}_{\boldsymbol{rr}})^{-\boldsymbol{1}}$. Generally, $\boldsymbol{B}^{\,r} \neq \boldsymbol{B}_{rr}$. It is noteworthy that the global equilibrium and local equilibria are simultaneously satisfied for the international input–output system.

Let us define country r's i-th value-added ratio as $v_{r(i)} = V_{r(i)}/X_{r(i)}$, where $V_{r(i)}$ is country r's i-th value-added. Country r's value-added ratio vector and the overall vector are $\boldsymbol{v}_r = (v_{r(i)})(1 \times n)$ and $\boldsymbol{v} = (\boldsymbol{v}_r)(1 \times (n \times R))$, respectively. Then, by virtue of definitions of input coefficients and value-added ratios, we have

$$\boldsymbol{u} = \boldsymbol{u}\boldsymbol{A} + \boldsymbol{v}; \;\; \boldsymbol{u}_n = \boldsymbol{u}_n\Sigma_k\boldsymbol{A}_{kr} + \boldsymbol{v}_r \;\;. \quad (5)$$

Therefore, value-added ratios are given by

$$\boldsymbol{v} = \boldsymbol{u}(\boldsymbol{I} - \boldsymbol{A});\ \boldsymbol{v}_r = \boldsymbol{u}_n(\boldsymbol{I}_n - \Sigma_k \boldsymbol{A}_{kr})\ . \tag{6}$$

where $\boldsymbol{u} =$ (1,1,…,1) (1×(n×R)) and $\boldsymbol{u}_n = (1,1, \dots ,1)$ (1×n) are aggregation vectors of unities. That is to say, the price vector associated with an input–output system in value terms always equals an aggregation vector.

3. Definitions and theorems for value-added trade

The new concept of value-added trade is defined as follows:

Definition 1. (Johnson and Noguera 2012; Kuboniwa 2015). The new concept of value-added exports and TiVA:

Using equation (2), country r's value-added exports to country s are defined as $\widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs}$, where $\widehat{\boldsymbol{V}}_r = diag\left\{v_{r_{(1)}}, \dots, v_{r_{(n)}}\right\}$ (n×n). The total value-added exports of origin country r to destination country s amount to $E^{va*}_{rs} = \boldsymbol{u}_n \widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs} = \boldsymbol{v}_r \boldsymbol{X}^*_{rs}$. At the non-bilateral level, the overall value-added exports of origin country r to all destination countries account for $E^{va*}_r = \boldsymbol{v}_r \sum_s \boldsymbol{X}^*_{rs}$.

Corresponding to Definition 1, we can define a country's factor content of trade employed worldwide to produce the country's net trade vector as follows:

Definition 2. (Trefler and Zhu 2010). The factor content of gross exports in the case with many countries and sectors:

We consider the following equation for country s's gross output vector $\boldsymbol{Z}_s$ employed to produce the country's net trade vector $\boldsymbol{E}_s^{net}$ for s = 1, 2, …,R (s ≠ r) in the case that country s exports to and imports from countries 1, …, r,…, R ($r \neq s$):

$$\boldsymbol{Z}_s = \boldsymbol{A}\boldsymbol{Z}_s + \boldsymbol{E}_s^{net} = \boldsymbol{B}\boldsymbol{E}_s^{net}, \tag{7}$$

where, assuming $s > r$ without loss of generality,

$$\boldsymbol{Z}_s = \begin{pmatrix} -\boldsymbol{X}_{1s}^{**} \\ \cdots \\ -\boldsymbol{X}_{rs}^{**} \\ \cdots \\ \boldsymbol{X}_{s1}^{**} + \cdots + \boldsymbol{X}_{sr}^{**} + \cdots + \boldsymbol{X}_{sR}^{**} \\ \cdots \\ -\boldsymbol{X}_{Rs}^{**} \end{pmatrix} \text{ and } \boldsymbol{E}_s^{net} = \begin{pmatrix} -\boldsymbol{E}_{1s} \\ \cdots \\ -\boldsymbol{E}_{rs} \\ \cdots \\ \boldsymbol{E}_{s1} + \cdots + \boldsymbol{E}_{sr} + \cdots + \boldsymbol{E}_{sR} \\ \cdots \\ -\boldsymbol{E}_{Rs} \end{pmatrix}.$$

$\boldsymbol{X}_{rs}^{**}$ $(n \times 1)$ is country r's gross output vector employed worldwide to produce the gross exports of country r to country s or the gross imports of country s from country r. Then, the factor content of gross exports from origin country r to destination country s is defined as $\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{**}$**.** The total factor content of gross exports from origin country r to destination country s amounts to $E_{rs}^{va**} = \boldsymbol{v}_r\boldsymbol{X}_{rs}^{**}$. Equation (7) simultaneously defines the overall factor content of origin country r to all destination countries, at the non-bilateral, which accounts for $E_r^{va**} = \boldsymbol{v}_r \sum_s \boldsymbol{X}_{rs}^{**}$.

We arrive at the following equivalence theorem:

Theorem 2. Bilateral equivalence theorem (Kuboniwa 2014b[2], 2015)

Definition 1 is bilaterally equivalent to Definition 2.

$\boldsymbol{X}_{rs}^{**} = \boldsymbol{X}_{rs}^{*}$, $\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{**} = \widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{*}$ and $\boldsymbol{v}_r\boldsymbol{X}_{rs}^{**} = \boldsymbol{v}_r\boldsymbol{X}_{rs}^{*}$; $E_{rs}^{va**} = E_{rs}^{va*}$ for r, s=1, 2,…., R (s≠r).

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2 Kuboniwa (2014b) proved the theorem for a 3-country world with many sectors.

Proof

In view of equations (2) and (3), we have

$$\boldsymbol{X}_{rs}^{*} = \Sigma_k \boldsymbol{A}_{rk}\boldsymbol{X}_{ks}^{*} + \boldsymbol{Y}_{rs} = (\Sigma_{k\neq s}\boldsymbol{A}_{rk}\boldsymbol{X}_{ks}^{*} + \boldsymbol{A}_{rs}\boldsymbol{X}_{ss}^{*}) + (\boldsymbol{E}_{rs} - \boldsymbol{A}_{rs}\boldsymbol{X}_{s}^{*})$$

$$= \Sigma_{k\neq s}\boldsymbol{A}_{rk}\boldsymbol{X}_{ks}^{*} - \boldsymbol{A}_{rs}\Sigma_{k\neq s}\boldsymbol{X}_{sk}^{*} + \boldsymbol{E}_{rs}. \tag{8}$$

On the other hand, it immediately follows from equation (7) that

$$\boldsymbol{X}_{rs}^{**} = \Sigma_{k\neq s}\boldsymbol{A}_{rk}\boldsymbol{X}_{ks}^{**} - \boldsymbol{A}_{rs}\Sigma_{k\neq s}\boldsymbol{X}_{sk}^{**} + \boldsymbol{E}_{rs}. \tag{9}$$

Let us compare equations (8) and (9). The solution for the equation system (2), $\boldsymbol{X}_{rs}^{*}$ can be a solution for the equation system (7), $\boldsymbol{X}_{rs}^{**}$, and vice versa. In view of the existence of the unique $\boldsymbol{B}$ due to productiveness of $\boldsymbol{A}$, we learn that the solution of equation (2) is unique and that the solution of equation (7) is also unique. Therefore, we can conclude

$$\boldsymbol{X}_{rs}^{*} \equiv \boldsymbol{X}_{rs}^{**} \text{ for all } r, s = 1, 2, \ldots, R\ (s\neq r). \tag{10}$$

This implies $\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{**} = \widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{*}$ and $\boldsymbol{v}_r\boldsymbol{X}_{rs}^{**} = \boldsymbol{v}_r\boldsymbol{X}_{rs}^{*}$ for *r*, *s*=1, 2,…., *R* (*s*≠*r*).

Q.E.D.

TiVA, which is based on value-added exports proposed by Johnson and Noguera (2012) and international organizations such as OECD and WTO, measures an origin (source) country's value-added employed worldwide to produce a destination country's final demand, excluding intermediates. The factor (value-added) content of trade, proposed by Trefler and Zhu (2010), measures a country's value-added employed worldwide to produce the country's net trade vector, which appropriately arranges gross exports and imports, including intermediates, as positive and negative elements, respectively. At a glance, these two measures may look quite different. However, we proved that in the world with many countries and sectors, these two measures of TiVA and the factor (value-added) content of trade are bilaterally equivalent.[3]

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[3] Johnson and Noguera (2012) implicitly suggest this equivalence. Foster-McGregor,

4. Decomposition of gross exports

4.1. The first stage:

First, we employ Koopman et al. (2010)'s decomposition of gross exports into domestic and foreign value-added terms. Making a matrix calculation of $\hat{\boldsymbol{V}}\boldsymbol{B}\hat{\boldsymbol{E}}$, where $\hat{\boldsymbol{V}}=diag\{\boldsymbol{v}_r\}$ and $\hat{\boldsymbol{E}}=diag\{\boldsymbol{E}_r\}$, its diagonal blocks and off-diagonal blocks yield domestic value-added and foreign value-added induced by gross exports, respectively. The domestic content (*DV*) and foreign content (*FV*) of value-added of country *r* are defined respectively as

$$DVr = \boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_r \text{ and } FVr = \Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr}\boldsymbol{E}_r. \qquad (11)$$

A country's gross exports generate its value-added. They also induce other countries' (foreign) value-added through the country's gross imports from many countries (many countries' exports to the country) to produce its gross exports. By virtue of the price equation (5) or (6), we have the following macro identity:

$$DVr + FVr = \boldsymbol{u}_n\boldsymbol{E}_r = Er. \qquad (12)$$

Thus a country's gross exports are fully decomposed into its domestic and foreign value-added contents induced by its gross exports. From equation (12) and definitions we always have

$$DVr \leq Er \text{ for all } r. \qquad (13)$$

Origin country's domestic content is always less than its gross exports by the foreign content. For convenience in the following discussion, we define country *r*'s domestic content induced by its exports to destination *s* as

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and Stehrer (2013) showed the non-bilateral equivalence, whereas they did not refer to the bilateral equivalence.

$$DV_{rs} = \boldsymbol{v}_r \boldsymbol{B}_{rr} \boldsymbol{E}_{rs}.$$

We also define country r's foreign content induced by its exports to destination s as

$$FV_{rs} = \Sigma_{k \neq r} \boldsymbol{v}_k \boldsymbol{B}_{kr} \boldsymbol{E}_{rs}.$$

In a similar way, we have

$$DVrs + FVrs = \boldsymbol{u}_n \boldsymbol{E}_{rs} = Ers. \tag{12'}$$

$$DVrs \leq Ers \text{ for all } r \text{ and } s \ (r \neq s). \tag{13'}$$

4.2. The second stage

The concept of value-added exports or TIVA clarified that a country's domestic DV_r should further be decomposed into value-added exports and part of double counts due to complicated foreign trade across countries. Koopman et al. (2010, 2014) linked Johnson and Noguera's definition of value-added exports with their concepts of domestic and foreign value-added. This implies that they linked "gross trade approach" to equation (11) with "final demand approach" to value-added exports (**Definition1**). Instead, we link "gross net trade approach" to equation (11) with our defined "gross net exports-approach" to value-added exports (**Definition 2**) so that we may directly and explicitly capture the relationships between value-added exports, domestic value-added and foreign value-added at the bilateral level by Definition 2 in a straightforward manner.

4.3. Two-country case

Let us consider two-country case (r,s =1,2) with many sectors. Our decomposition result on the first stage and the second stage is shown by Figure 1.

[Figure 1 here]

The first stage:

For country 1: In view of equation (11) we have the following results.

The total value of gross exports is $E_1 = E_{12} = \boldsymbol{u}_n \boldsymbol{E}_{12}$.

The domestic content is $DV_1 = \boldsymbol{v}_1 \boldsymbol{B}_{11} \boldsymbol{E}_{12}$.

The foreign content is $FV_1 = \boldsymbol{v}_2 \boldsymbol{B}_{21} \boldsymbol{E}_{12}$.

This foreign content returns to country 2.

For country 2: Similarly, we have the following.

The total value of gross exports is $E_2 = E_{21} = \boldsymbol{u}_n \boldsymbol{E}_{21}$.

The domestic content is $DV_2 = \boldsymbol{v}_2 \boldsymbol{B}_{22} \boldsymbol{E}_{21}$.

The foreign content is $FV_2 = \boldsymbol{v}_1 \boldsymbol{B}_{12} \boldsymbol{E}_{21}$.

This foreign content returns to country 1.

The second stage:

For country 1: By virtue of equation (7), we have the following result.

The total value of value-added exports is $E_1^{va} = E_{12}^{va} = \boldsymbol{v}_1(\boldsymbol{B}_{11}\boldsymbol{E}_{12} - \boldsymbol{B}_{12}\boldsymbol{E}_{21})$.

The difference between DV_1 and E_1^{va} equals the value-added which returns home from country 2, $\boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{21}$. This means that the double count in the domestic content equals the value-added which returns home.

For country 2: Similarly, we have the following.

The total value of value-added exports is $E_2^{va} = E_{21}^{va} = \boldsymbol{v}_2(\boldsymbol{B}_{22}\boldsymbol{E}_{21} - \boldsymbol{B}_{21}\boldsymbol{E}_{12})$.

The difference between DV_2 and E_2^{va} equals the value-added which returns home from country 1, $\boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{12}$. This implies that the double account in the domestic content equals the value-added which returns home.

In a 2-country case with many sectors, first, we do not have to distinguish the bilateral exports from each country’s total exports. Second, the double counting in the home domestic content of gross exports is merely the total value-added of the foreign content, which is produced in the corresponding destination partner (the rest of the world) for origin's gross exports and returns home. That is to say, all of home value-added demanded by the partner for home's gross exports should constitute the double counting in the home domestic content. Third, the bilateral total value of origin country's value-added exports is always less than its domestic content by the double count returned home. Fourth, in our two-country case, if country 1 is a specific country such as China, country 2 should be the rest of the world, including all other countries such as the USA, Japan and others. We cannot use our two-country case assuming that country 1 is China and country 2 is only the USA.[4]

Accounting of gross exports for a two-country case

Summarizing the above discussions, gross exports are decomposed as follows:

$$E_{12} = DV_1+FV_1 = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{12} + \boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{12}$$

$$= E_{12}^{va} + \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{21} + FV_1 = \boldsymbol{v}_1(\boldsymbol{B}_{11}\boldsymbol{E}_{12} - \boldsymbol{B}_{12}\boldsymbol{E}_{21}) + [\boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{21} + \boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{12}]. \tag{14}$$

$$E_{21} = DV_2+FV_2 = \boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_{21} + \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{21}$$

$$= E_{21}^{va} + \boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{12} + FV_2 = \boldsymbol{v}_2(\boldsymbol{B}_{22}\boldsymbol{E}_{21} - \boldsymbol{B}_{21}\boldsymbol{E}_{12}) + [\boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{12} + \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{21}]. \tag{15}$$

Terms of [.] show total double counts in the gross exports. Equations (14) and (15) are essentially equivalent to Koopman et al. (2014)'s equations 11, 13 and 14 whereas our equations straightforwardly present double counted elements and their sources by definition without any toil and tear. Equation 11 in Koopman et al. (2014) can be arranged as

$$E_{12} = VT_{12} + \boldsymbol{v}_1\boldsymbol{B}_{12}(\boldsymbol{A}_{21}\boldsymbol{X}_1 + \boldsymbol{Y}_{21}) + \boldsymbol{v}_2\boldsymbol{B}_{21}(\boldsymbol{A}_{12}\boldsymbol{X}_2 + \boldsymbol{Y}_{12}), \tag{16}$$

where $VT_{12} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{Y}_{12} + \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{Y}_{22}$ is country 1's value-added exports by Johnson and

4 In this context, the numerical example of two-country case (China and the USA) in Koopman et al. (2014) may be inappropriate.

Noguera's definition (Definition1). By virtue of Theorem 1, $VT_{12}= E_{12}^{va}$, and $\boldsymbol{E}_{21} = \boldsymbol{A}_{21}\boldsymbol{X}_1 + \boldsymbol{Y}_{21}$ and $\boldsymbol{E}_{12} = \boldsymbol{A}_{12}\boldsymbol{X}_2 + \boldsymbol{Y}_{12}$. Thus, the above equation comes down to our equation (14). However, as the double counting part is hided in Definition 1, we may toil up to equation (16).

In the two-country case, from equations (14) and (15), a country's gross trade balance with the rest of the world or net 'gross exports' results in

$$T_{12}^{g} = E_{12} - E_{21} = E_{12}^{va} - E_{21}^{va} = T_{12}^{va};$$

$$T_{21}^{g} = E_{12} - E_{21} = E_{12}^{va} - E_{21}^{va} = T_{21}^{va}.$$

At the macro-economy level, generally, a country's gross trade balance is identical to its value-added trade balance (Kuboniwa 2015, Theorem 1 and Theorem 1'). That is to say, double accounting parts of gross exports vanish in GDP on the expenditure side.[5]

Country r's GDP_r at the expenditure side is as follows:

$$GDP_1 = \boldsymbol{u}_n(\boldsymbol{Y}_{11} + \boldsymbol{Y}_{21} + \boldsymbol{E}_{12} - \boldsymbol{E}_{21}),$$

$$GDP_2 = \boldsymbol{u}_n(\boldsymbol{Y}_{12} + \boldsymbol{Y}_{22} + \boldsymbol{E}_{21} - \boldsymbol{E}_{12}).$$

By virtue of well-known Leontief's duality equation with a price system $\boldsymbol{p} = \boldsymbol{u}_n$, GDP at the production side is identical to GDP at the expenditure side.[6] In our framework, we can represent the following:

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5 Using local equilibrium equation (4), Koopman et al. further decompose intermediate imports, $\boldsymbol{A}_{sr}\boldsymbol{X}_r$ (country r) of equation (16) into $\boldsymbol{A}_{sr}\boldsymbol{B}^r\boldsymbol{Y}_{rr}$ and $\boldsymbol{A}_{sr}\boldsymbol{B}^r\boldsymbol{E}_{rs}$. Then they expand country r's GDP ($GDP_r = VT_r + v_r\boldsymbol{X}_{rr}$) based on equation (2) and inversion by 4-block partitioning. Comparing their decomposition and GDP concept, they find a 'pure double-counted term' of double-counted terms, $\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{A}_{sr}\boldsymbol{B}^r\boldsymbol{E}_{rs}$ for country r. Their fact-finding may need further investigation. However, in view of our discussion based on GDP on the expenditure side, their finding may not be significant for gross exports accounting.

6 For example, see Miller and Blair (2009, p.53).

GDP_1=$\boldsymbol{v}_1\boldsymbol{X}_1 = \boldsymbol{v}_1(\boldsymbol{X}_{11} + \boldsymbol{X}_{12}) = E_{12}^{va} + \boldsymbol{v}_1\boldsymbol{X}_{11}$,

GDP_2=$\boldsymbol{v}_2\boldsymbol{X}_2 = \boldsymbol{v}_2(\boldsymbol{X}_{21} + \boldsymbol{X}_{22}) = E_{21}^{va} + \boldsymbol{v}_2\boldsymbol{X}_{22}$.

4.4. A three-country case

Next, we examine a three-country case (r,s =1, 2, 3) with many sectors. Our decomposition results at the first and the second stages are shown by Figure 2.

[Figure 2 here]

The first stage:

For country 1: It follows from equation (18) that:

The total value of gross exports is E_1=$\boldsymbol{u}_n(\boldsymbol{E}_{12} + \boldsymbol{E}_{13})$.

The domestic content is $DV_1 = \boldsymbol{v}_1\boldsymbol{B}_{11}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13})$.
The foreign content is FV_1= $\boldsymbol{v}_2\boldsymbol{B}_{21}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13})$+$\boldsymbol{v}_3\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13})$.
Unlike two-country case, a part of this foreign content does not return to countries 2 and 3. This reason will be clarified when we examine the bilateral level.

For country 2: Similarly, we have:

The total value of gross exports is $E_2 = \boldsymbol{u}_n(\boldsymbol{E}_{21} + \boldsymbol{E}_{23})$.

The domestic content is $DV_2 = \boldsymbol{v}_2\boldsymbol{B}_{22}(\boldsymbol{E}_{21}+ \boldsymbol{E}_{23})$.
The foreign content is $FV_2 = \boldsymbol{v}_1\boldsymbol{B}_{12}(\boldsymbol{E}_{21}+ \boldsymbol{E}_{23})$+$\boldsymbol{v}_3\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+ \boldsymbol{E}_{23})$.
A part of this foreign content does not return to countries 1 and 3.

For country 3: We also have:

The total value of gross exports is $E_3 = \boldsymbol{u}_n(\boldsymbol{E}_{31} + \boldsymbol{E}_{32})$. .

The domestic content is $DV_3 = \boldsymbol{v}_3\boldsymbol{B}_{33}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32})$.
The foreign content is $FV_3 = \boldsymbol{v}_1\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32})$+$\boldsymbol{v}_2\boldsymbol{B}_{23}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32})$.
A part of this foreign content does not appear to return to countries 1 and 2.

The second stage:

For country 1:

At the non-bilateral level:

From equation (11), the total value of value-added exports is

$$E_1^{va} = \boldsymbol{v}_1\{\boldsymbol{B}_{11}(\boldsymbol{E}_{12} + \boldsymbol{E}_{13}) - \boldsymbol{B}_{12}\boldsymbol{E}_{21} - \boldsymbol{B}_{13}\boldsymbol{E}_{31}\}.$$

The difference between DV_1 and E_1^{va} equals the value-added which returns home from country 2 and 3, $\boldsymbol{v}_1(\boldsymbol{B}_{12}\boldsymbol{E}_{21} + \boldsymbol{B}_{13}\boldsymbol{E}_{31})$. This difference shows the double count in the domestic content. Unlike 2-country case, capturing double counts for 3-country case is rather complicated. At the non-bilateral level, a part of foreign contents (FV_2 and FV_3) generated by country 1 and country 2, $\boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{23} + \boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_{32}$, does not return home as double counts (no-return as double counts). However, as will be shown soon, with an adverse adjustment, this part also returns home at the bilateral level. By definitions, we have always $DV_1 \geq E_1^{va}$. Origin country's domestic content is always greater than its value-added exports by the double count returned home.

At the bilateral level:

In view of equation (7), the value of country 1's value-added exports to country 2 is defined as

$$E_{12}^{va} = \boldsymbol{v}_1\{\boldsymbol{B}_{11}\boldsymbol{E}_{12} - \boldsymbol{B}_{12}(\boldsymbol{E}_{21} + \boldsymbol{E}_{23}) + \boldsymbol{B}_{13}\boldsymbol{E}_{32}\}.$$

A part of the difference between $DV_{12} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{12}$ of DV_1 and E_{12}^{va} equals the foreign value-added which returns home from country 2, $\boldsymbol{v}_1\boldsymbol{B}_{12}(\boldsymbol{E}_{21} + \boldsymbol{E}_{23})$ as the double counts in the domestic content. The term of $\boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_{32}$, which is country 1's value-added generated by non-destination country 3's exports to destination country 2 (country 2's imports from country 3), is a part of the foreign content of non-destination country 3 for origin country 1. At the bilateral level, all the foreign content of the destination country for the origin country returns home as double counts. However, this needs an adverse adjustment of the foreign content of the non-destination country for the origin country. The adjustment term returns home not as the double accounting but as the additive count for the value-added exports (no return as double counts but return as additive counts).

It is noteworthy to learn the following fact:

$$DV_{12} \gtreqless E_{12}^{va} \text{ if } \boldsymbol{v}_1\boldsymbol{B}_{12}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23}) \gtreqless \boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_{32}.$$

The domestic content of origin country's value-added induced by its exports to the destination country can be greater (less) than its value-added exports if the double count returned home is greater (less) than the additive term returned home.

The value of country 1's value-added exports to country 3 is defined as

$$E_{13}^{va} = \boldsymbol{v}_1\{\boldsymbol{B}_{11}\boldsymbol{E}_{13}-\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})+\boldsymbol{B}_{12}\boldsymbol{E}_{23}\}].$$

A part of the difference between the term $DV_{13} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{13}$ of DV_1 and E_{13}^{va} equals the value-added which returns home from country 3, $\boldsymbol{v}_1\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})$, which is the foreign content of country 3 for country 1and the double count part in the domestic content at the bilateral level. The term of $\boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{23}$, which is country 1's value-added generated by non-destination country 2's exports to destination country 3 (country 3's imports from country 2), which is a part of the foreign content of country 2 for country 1. This part of the foreign content returns home as not the double count but the additive count to the bilateral value-added exports.

As can easily be verified, we have

$$E_{12}^{va}+E_{13}^{va} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{12}-\boldsymbol{B}_{12}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23})+\boldsymbol{B}_{13}\boldsymbol{E}_{32}\} + \boldsymbol{v}_1\{\boldsymbol{B}_{11}\boldsymbol{E}_{13}-\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})+\boldsymbol{B}_{12}\boldsymbol{E}_{23}\}$$

$$= \boldsymbol{v}_1\{\boldsymbol{B}_{11}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})-\boldsymbol{B}_{12}\boldsymbol{E}_{21}-\boldsymbol{B}_{13}\boldsymbol{E}_{31}\} = E_1^{va}.$$

In E_1^{va}, terms across countries 2 and 3 for adjustments, $\boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_{32}$ and $\boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{23}$ in E_{12}^{va} and E_{13}^{va} are cancelled out.

For country 2:

At the non-bilateral level:

Similarly, the total value of value-added exports is

$$E_2^{va} = \boldsymbol{v}_2\{\boldsymbol{B}_{22}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23})-\boldsymbol{B}_{21}\boldsymbol{E}_{12}-\boldsymbol{B}_{23}\boldsymbol{E}_{32}\}$$

The difference between DV_2 and E_2^{va} equals the value-added which returns home from country 2 and 3, $\boldsymbol{v}_2(\boldsymbol{B}_{21}\boldsymbol{E}_{12}+\boldsymbol{B}_{23}\boldsymbol{E}_{32})$. This difference demonstrates the double count in the domestic content.

At the bilateral level:

The value of country 2's value-added exports to country 1 is calculated by

$$E_{21}^{va} = \boldsymbol{v}_2\{\boldsymbol{B}_{22}\boldsymbol{E}_{21}-\boldsymbol{B}_{21}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})+\boldsymbol{B}_{23}\boldsymbol{E}_{31}\}.$$

A part of difference between the term $DV_{21}=\boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_{21}$ of DV_2 and E_{12}^{va} equals the value-added which returns home from country 1, $\boldsymbol{v}_2\{\boldsymbol{B}_{21}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})$, which constitutes the double count in the domestic content of country 2 for country 1. The term of $\boldsymbol{v}_2\boldsymbol{B}_{23}\boldsymbol{E}_{31}$, which is country 2's value-added generated by non-destination country 3's exports to destination country 1 (country 1's imports from country 3), is a part of the foreign content of country 3 for country 2. This term returns home as the additive count.

The value of country 2's value-added exports to country 3 is

$$E_{23}^{va} = \boldsymbol{v}_2\{\boldsymbol{B}_{22}\boldsymbol{E}_{23}-\boldsymbol{B}_{23}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})+\boldsymbol{B}_{21}\boldsymbol{E}_{13}\}.$$

A part of the difference between the term $\boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_{23}$ of DV_2 and E_{23}^{va} equals the value-added which returns home from country 3 with an adjustment, $\boldsymbol{v}_2\boldsymbol{B}_{23}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})$, which constitutes the double count in the domestic content of country 2 for country 3. The term of $\boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{13}$, is a part of the foreign content of country 1 for country 2.

In a similar way, we have $E_2^{va} = E_{21}^{va} + E_{23}^{va}$.

For country 3:

At the non-bilateral level:

The total value of value-added exports is

$$E_3^{va} = \boldsymbol{v}_3\{\boldsymbol{B}_{33}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32})-\boldsymbol{B}_{31}\boldsymbol{E}_{13}-\boldsymbol{B}_{32}\boldsymbol{E}_{23}\}.$$

The difference between $DV_3$and E_3^{va} equals the value-added which returns home from country 1 and 2, $\boldsymbol{v}_3(\boldsymbol{B}_{31}\boldsymbol{E}_{13}+\boldsymbol{B}_{32}\boldsymbol{E}_{23})$. This difference provides the double count in the domestic content.

At the bilateral level:

The value of country 3's value-added exports to country 1 is

$$E_{31}^{va} = \boldsymbol{v}_3\{\boldsymbol{B}_{33}\boldsymbol{E}_{31}-\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})+\boldsymbol{B}_{32}\boldsymbol{E}_{21}\}.$$

The difference between the term $DV_{31}=\boldsymbol{v}_3\boldsymbol{B}_{33}\boldsymbol{E}_{31}$ of DV_3 and E_{31}^{va} equals the value-added which returns home from country 1 with an adjustment, $\boldsymbol{v}_3\{\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})-\boldsymbol{B}_{32}\boldsymbol{E}_{21}\}$ which yields the double count part in the domestic content of country 1 for country 3. The term of $\boldsymbol{v}_3\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13})$ is the foreign content of country 1 for country 3. The term of $\boldsymbol{v}_3\boldsymbol{B}_{32}\boldsymbol{E}_{21}$, which is country 3's value-added generated by country 2's exports to country 1 (country 1's imports from country 2), is a part of the foreign content of country 2 for country 3.

The value of country 3's value-added exports to country 2 is

$$E_{32}^{va} = \boldsymbol{v}_3\{\boldsymbol{B}_{33}\boldsymbol{E}_{32}-\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23})+\boldsymbol{B}_{31}\boldsymbol{E}_{12}\}.$$

The difference between the term $DV_{32}=\boldsymbol{v}_3\boldsymbol{B}_{33}\boldsymbol{E}_{32}$ of DV_3 and E_{32}^{va} equals the value-added which returns home from country 2 with an adjustment, $\boldsymbol{v}_3\{\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23})-\boldsymbol{B}_{31}\boldsymbol{E}_{12}\}$, which is the double count part in the domestic content of country 3 for country 2. The term of $\boldsymbol{v}_3\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23})$ is the foreign content of country 2 for country 3. The term of $\boldsymbol{v}_3\boldsymbol{B}_{31}\boldsymbol{E}_{12}$, which is country 3's value-added generated by non-destination country 1's exports to destination country 2 (country 2's imports from country 1), is a part of the foreign content of country 1 for country 3.

Similarly, we have $E_3^{va} = E_{31}^{va} + E_{32}^{va}$.

Accounting of gross exports for three-country case

Summarizing the above discussions, gross exports are decomposed as follows:

At the non-bilateral level:

$E_1 = E_{12} + E_{13} = DV_1 + FV_1 = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_1 + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_1$

$= E_1^{va} + \boldsymbol{v}_1(\boldsymbol{B}_{12}\boldsymbol{E}_{21} + \boldsymbol{B}_{13}\boldsymbol{E}_{31}) + FV_1$

$= \boldsymbol{v}_1(\boldsymbol{B}_{11}\boldsymbol{E}_1 - \boldsymbol{B}_{12}\boldsymbol{E}_{21} - \boldsymbol{B}_{13}\boldsymbol{E}_{31}) + [\boldsymbol{v}_1(\boldsymbol{B}_{12}\boldsymbol{E}_{21} + \boldsymbol{B}_{13}\boldsymbol{E}_{31}) + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_1]$.

$E_2 = E_{21} + E_{23} = DV_2 + FV_2 = \boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_2 + (\boldsymbol{v}_1\boldsymbol{B}_{12} + \boldsymbol{v}_3\boldsymbol{B}_{32})\boldsymbol{E}_2$

$= E_2^{va} + \boldsymbol{v}_2(\boldsymbol{B}_{21}\boldsymbol{E}_{12} + \boldsymbol{B}_{21}\boldsymbol{E}_{32}) + FV_2$

$= \boldsymbol{v}_2(\boldsymbol{B}_{22}\boldsymbol{E}_2 - \boldsymbol{B}_{21}\boldsymbol{E}_{12} - \boldsymbol{B}_{21}\boldsymbol{E}_{32}) + [\boldsymbol{v}_2(\boldsymbol{B}_{21}\boldsymbol{E}_{12} + \boldsymbol{B}_{21}\boldsymbol{E}_{32}) + (\boldsymbol{v}_1\boldsymbol{B}_{12} + \boldsymbol{v}_3\boldsymbol{B}_{32})\boldsymbol{E}_2]$.

$E_3 = E_{31} + E_{32} = DV_3 + FV_3 = \boldsymbol{v}_3\boldsymbol{B}_{33}\boldsymbol{E}_3 + (\boldsymbol{v}_1\boldsymbol{B}_{13} + \boldsymbol{v}_2\boldsymbol{B}_{23})\boldsymbol{E}_3$

$= E_3^{va} + \boldsymbol{v}_3(\boldsymbol{B}_{31}\boldsymbol{E}_{13} + \boldsymbol{B}_{32}\boldsymbol{E}_{23}) + FV_3$

$= \boldsymbol{v}_3(\boldsymbol{B}_{33}\boldsymbol{E}_3 - \boldsymbol{B}_{31}\boldsymbol{E}_{13} - \boldsymbol{B}_{32}\boldsymbol{E}_{23}) + [\boldsymbol{v}_3(\boldsymbol{B}_{31}\boldsymbol{E}_{13} + \boldsymbol{B}_{32}\boldsymbol{E}_{23}) + (\boldsymbol{v}_1\boldsymbol{B}_{13} + \boldsymbol{v}_2\boldsymbol{B}_{23})\boldsymbol{E}_3]$.

Terms of [.] show total double counts in the gross exports.

At the bilateral level:

Assume that origin/source is country 1, the destination country is 2 or 3 and the non-destination/third party country is 3 or 2.

$E_{12} = DV_{12} + FV_{12} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{12} + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{12}$

$= E_{12}^{va} + \boldsymbol{v}_1\{\boldsymbol{B}_{12}(\boldsymbol{E}_{21} + \boldsymbol{E}_{23}) - \boldsymbol{B}_{13}\boldsymbol{E}_{32}\} + FV_{12}$

$= \boldsymbol{v}_1\{\boldsymbol{B}_{11}\boldsymbol{E}_{12} - \boldsymbol{B}_{12}(\boldsymbol{E}_{21} + \boldsymbol{E}_{23}) + \boldsymbol{B}_{13}\boldsymbol{E}_{32}\} + [\boldsymbol{v}_1\{\boldsymbol{B}_{12}(\boldsymbol{E}_{21} + \boldsymbol{E}_{23}) - \boldsymbol{B}_{13}\boldsymbol{E}_{32}\} + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{12}]$.

$E_{13} = DV_{13} + FV_{13} = \boldsymbol{v}_1\boldsymbol{B}_{11}\boldsymbol{E}_{13} + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{13}$

$= E_{13}^{va} + \boldsymbol{v}_1\{\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32}) - \boldsymbol{B}_{12}\boldsymbol{E}_{23}\} + FV_{13}$

$= \boldsymbol{v}_1\{\boldsymbol{B}_{11}\boldsymbol{E}_{13} - \boldsymbol{B}_{13}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32}) + \boldsymbol{B}_{12}\boldsymbol{E}_{23}\} + [\boldsymbol{v}_1\{\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32}) - \boldsymbol{B}_{12}\boldsymbol{E}_{23}\} + (\boldsymbol{v}_2\boldsymbol{B}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{12}]$.

Suppose that origin/source is country 2, the destination country is 1 or 3 and the non-destination/third party country is 3 or 1.

$E_{21} = DV_{21}+FV_{21} = \boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_{21} + (\boldsymbol{v}_1\boldsymbol{B}_{12}+ \boldsymbol{v}_3\boldsymbol{B}_{32})\boldsymbol{E}_{21}$

$= E_{21}^{va} + \boldsymbol{v}_2\{\boldsymbol{B}_{21}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13}) - \boldsymbol{B}_{23}\boldsymbol{E}_{31}\} + FV_{21}$

$= \boldsymbol{v}_2\{\boldsymbol{B}_{22}\boldsymbol{E}_{21} - \boldsymbol{B}_{21}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13}) + \boldsymbol{B}_{23}\boldsymbol{E}_{31}\} + [\boldsymbol{v}_2\{\boldsymbol{B}_{21}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13}) - \boldsymbol{B}_{23}\boldsymbol{E}_{31}\} + (\boldsymbol{v}_1\boldsymbol{B}_{12} + \boldsymbol{v}_3\boldsymbol{B}_{32})\boldsymbol{E}_{21}]$.

$E_{23} = DV_{23}+FV_{23} = \boldsymbol{v}_2\boldsymbol{B}_{22}\boldsymbol{E}_{23} + (\boldsymbol{v}_1\boldsymbol{B}_{21}+ \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{23}$

$= E_{23}^{va} + \boldsymbol{v}_2\{\boldsymbol{B}_{23}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32}) - \boldsymbol{B}_{21}\boldsymbol{E}_{13}\} + FV_{23}$

$= \boldsymbol{v}_2\{\boldsymbol{B}_{22}\boldsymbol{E}_{23} - \boldsymbol{B}_{23}(\boldsymbol{E}_{31}+ \boldsymbol{E}_{32}) + \boldsymbol{B}_{21}\boldsymbol{E}_{13}\} + [\boldsymbol{v}_2\{\boldsymbol{B}_{13}(\boldsymbol{E}_{31}+\boldsymbol{E}_{32}) - \boldsymbol{B}_{12}\boldsymbol{E}_{23}\} + (\boldsymbol{v}_1\boldsymbol{B}_{12} + \boldsymbol{v}_3\boldsymbol{B}_{32})\boldsymbol{E}_{23}]$.

Let origin/source be country 3, the destination country be 1 or 2 and the non-destination/third party country be 2 or 1.

$E_{31} = DV_{31}+FV_{32} = \boldsymbol{v}_3\boldsymbol{B}_{33}\boldsymbol{E}_{31} + (\boldsymbol{v}_1\boldsymbol{B}_{13}+ \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{31}$

$= E_{31}^{va} + \boldsymbol{v}_3\{\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13}) - \boldsymbol{B}_{32}\boldsymbol{E}_{21}\} + FV_{31}$

$= \boldsymbol{v}_3\{\boldsymbol{B}_{33}\boldsymbol{E}_{31} - \boldsymbol{B}_{31}(\boldsymbol{E}_{12}+ \boldsymbol{E}_{13}) + \boldsymbol{B}_{32}\boldsymbol{E}_{21}\} + [\boldsymbol{v}_3\{\boldsymbol{B}_{31}(\boldsymbol{E}_{12}+\boldsymbol{E}_{13}) - \boldsymbol{B}_{32}\boldsymbol{E}_{21}\} + (\boldsymbol{v}_1\boldsymbol{B}_{13} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{31}]$.

$E_{32} = DV_{32}+FV_{32} = \boldsymbol{v}_3\boldsymbol{B}_{33}\boldsymbol{E}_{32} + (\boldsymbol{v}_1\boldsymbol{B}_{13}+ \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{32}$

$= E_{32}^{va} + \boldsymbol{v}_3\{\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+ \boldsymbol{E}_{23}) - \boldsymbol{B}_{31}\boldsymbol{E}_{12}\} + FV_{32}$

$= \boldsymbol{v}_3\{\boldsymbol{B}_{33}\boldsymbol{E}_{32} - \boldsymbol{B}_{32}(\boldsymbol{E}_{21}+ \boldsymbol{E}_{23}) + \boldsymbol{B}_{31}\boldsymbol{E}_{12}\} + [\boldsymbol{v}_3\{\boldsymbol{B}_{32}(\boldsymbol{E}_{21}+\boldsymbol{E}_{23}) - \boldsymbol{B}_{31}\boldsymbol{E}_{12}\} + (\boldsymbol{v}_1\boldsymbol{B}_{13} + \boldsymbol{v}_3\boldsymbol{B}_{31})\boldsymbol{E}_{32}]$.

$+ \boldsymbol{v}_3\, \boldsymbol{B}_{32}\boldsymbol{E}_{21} + \boldsymbol{v}_3\, \boldsymbol{B}_{31}\boldsymbol{E}_{12}$

At the bilateral level, the overall sum of return home from each destination country as double accounts, $R(-)$ which should be deducted from value-added exports, amount to:

$RETURN(-) = \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_1 + \boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_3 + \boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_1 + \boldsymbol{v}_2\boldsymbol{B}_{23}\boldsymbol{E}_3 + \boldsymbol{v}_3\boldsymbol{B}_{31}\boldsymbol{E}_1 + \boldsymbol{v}_3\boldsymbol{B}_{32}\boldsymbol{E}_2 = FV_1+FV_2+FV_3$.

At the bilateral level, all foreign contents return origin home as double counts deducted from the origin's value-added exports. However, at the same time, additive counts return home from a non-destination or third party, the total of which accounts for

$RETURN(+) = \boldsymbol{v}_1\boldsymbol{B}_{13}\boldsymbol{E}_{32} + \boldsymbol{v}_1\boldsymbol{B}_{12}\boldsymbol{E}_{23} + \boldsymbol{v}_2\boldsymbol{B}_{23}\boldsymbol{E}_{31} + \boldsymbol{v}_2\boldsymbol{B}_{21}\boldsymbol{E}_{13} + \boldsymbol{v}_3\boldsymbol{B}_{32}\boldsymbol{E}_{21} + \boldsymbol{v}_3\boldsymbol{B}_{31}\boldsymbol{E}_{12}$.

These are part of foreign contents. At the non-bilateral level, this results in the total of double accounts returning home less than the total foreign content.

4.5. The general case

We are now in a position to present the general case with many countries and many sectors as follows:

The first stage:

For an arbitrary country, r (1,2,3,…,R)**:** It follows from equation (11) that:

Gross exports are given by equation (3) as $\boldsymbol{E}_r = \Sigma_{s\neq r}\boldsymbol{E}_{rs}$.
The total value of gross exports is $E_r = \boldsymbol{u}_n\boldsymbol{E}_r$.
The domestic content is $DV_r = \boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_r$.
The foreign content is $FV_r = \Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr}\boldsymbol{E}_r$

The second stage:

For country *r*:

At the non-bilateral level:

By virtue of equation (7), the total value of country r's value-added exports is

$$E_r^{va} = \boldsymbol{v}_r(\boldsymbol{B}_{rr}\boldsymbol{E}_r - \Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr}). \quad (17)$$

The difference between DV_r and E_r^{va} equals the value-added which returns home from the world (countries $s \neq r$), $\boldsymbol{v}_r\Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr}$. This difference shows the double accounting in the domestic content. The foreign content of each partner s is defined as

$$FV_s = \Sigma_{r\neq s}\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{E}_s = \Sigma_{r\neq s}\boldsymbol{v}_r\boldsymbol{B}_{rs}(\boldsymbol{E}_{s1} + \boldsymbol{E}_{s2} + \boldsymbol{E}_{s3} + \cdots + \boldsymbol{E}_{sr} + \cdots + \boldsymbol{E}_{sR}).$$

As can be seen, only one part of r's value-added generated by foreign partner s's exports, $\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{E}_{sr}$ returns home r as a double accounting term.

At the bilateral level:

In view of equation (7), the value of origin country *r*'s value-added exports to destination country $s \neq \boldsymbol{r}$ is defined as

$$E_{rs}^{va} = \boldsymbol{v}_r\{\boldsymbol{B}_{rr}\boldsymbol{E}_{rs} - \boldsymbol{B}_{rs}\boldsymbol{E}_s + \Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}\}. \tag{18}$$

A part of the difference between $DV_{rs} = \boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_{rs}$ of DV_r and E_{rs}^{va} equals the value-added which returns home from country *s*, $\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{E}_s$. This difference constitutes the double count in the domestic content. The term of $\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{E}_s$ is clearly country *r*'s foreign value-added generated by destination country's gross exports to the world. The term of $\boldsymbol{v}_r\Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}$, which is origin country *r*'s value-added generated by the third party non-destination country *k*'s exports to destination country *s* (country *s*'s imports from country *k*), is a part of the foreign content of country *k* for country *r*. At the bilateral level, all the foreign content of the destination country for the origin country returns home. However, this needs an adverse adjustment. The adverse adjustment term is the foreign content of the third party non-destination countries for the origin country which returns origin home not as the double count deducted from the value-added exports but as the positive addition to the value-added exports.

As can easily be seen, we have $E_r^{va} = \Sigma_{s\neq r}E_{rs}^{va}$. Clearly, in view of equation (21) and (22), and definitions, we also have the following theorem.

Theorem 2. For $r = 1, 2, 3, \ldots, R$

$$DV_r \geq E_r^{va};$$

$$DV_{rs} \gtreqless E_{rs}^{va} \text{ if } v_r\boldsymbol{B}_{rs}\boldsymbol{E}_s \lesseqgtr v_r\Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}\,.\ (s = 1, 2, 3, \ldots, R;\ R \geqq 3;\ s \neq r) \tag{19}$$

Origin country r's domestic value-added content induced by its exports to all other destination countries is always greater than its value-added exports to all other countries by the double count, $\boldsymbol{v}_r\Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr}$, which is the sum of r's value-added induced by destination country s's exports to origin country r, over s. Origin country r's domestic value-added content induced by its exports to a destination country s can be greater (less) than its value-added exports if the double count returned home r from the destination s is greater (less) than the additive count returned home r from all other non-destination countries k.

Remark:

It would be an easy exercise to extend our discussions by dividing gross exports into two parts, that is to say, exports for intermediate demand and for final demand.

Accounting of gross exports for the general case

$$E_r = \Sigma_{s\neq r}E_{rs} = DV_r + FV_r = \boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_r + (\Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr})\boldsymbol{E}_r$$

$$= E_r^{va} + \boldsymbol{v}_r\Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr} + FV_r$$

$$= \boldsymbol{v}_r(\boldsymbol{B}_{rr}\boldsymbol{E}_r - \Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr}) + [\boldsymbol{v}_r\Sigma_{s\neq r}\boldsymbol{B}_{rs}\boldsymbol{E}_{sr} + (\Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr})\boldsymbol{E}_r]. \quad (20)$$

Terms of [.] show total double counts in the gross exports.

As $\boldsymbol{E}_{sr} = \boldsymbol{A}_{sr}\boldsymbol{X}_r + \boldsymbol{Y}_{sr}$ $(s \neq r)$, $\boldsymbol{E}_r = \Sigma_{s\neq r}\boldsymbol{E}_{rs} = \Sigma_{s\neq r}(\boldsymbol{A}_{rs}\boldsymbol{X}_s + \boldsymbol{Y}_{rs})$ and $E_r^{va} = VT_r$, the above equation can be written as

$$E_r = VT_r + [\boldsymbol{v}_r\Sigma_{s\neq r}\boldsymbol{B}_{rs}(\boldsymbol{A}_{sr}\boldsymbol{X}_r + \boldsymbol{Y}_{sr}) + (\Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr})\{\Sigma_{s\neq r}(\boldsymbol{A}_{rs}\boldsymbol{X}_s + \boldsymbol{Y}_{rs})\}]. \quad (21)$$

This is exactly equation 34 in Koopman et al. (2015, p.480).

$$E_{rs}=DV_{rs}+FV_{rs} = \boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_{rs} + (\Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr})\boldsymbol{E}_{rs}$$

$$= E_{rs}^{va} + \boldsymbol{v}_r(\boldsymbol{B}_{rs}\boldsymbol{E}_s+\Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}) + FV_r$$

$$=(\boldsymbol{v}_r\boldsymbol{B}_{rr}\boldsymbol{E}_r - \boldsymbol{B}_{rs}\boldsymbol{E}_s+\Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}) + [\boldsymbol{v}_r(\boldsymbol{B}_{rs}\boldsymbol{E}_s - \Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}) + (\Sigma_{s\neq r}\boldsymbol{v}_s\boldsymbol{B}_{sr})\boldsymbol{E}_r]. \quad (22)$$

At the bilateral level, the overall sum of return home from each destination country as double accounts, *RETURN*(-) which should be deducted from value-added exports, amount to:

$$RETURN(-) = \Sigma_{r,s,s\neq r}\boldsymbol{v}_r(\boldsymbol{B}_{rs}\boldsymbol{E}_s) = \Sigma_r FV_r. \quad (23)$$

At the bilateral level, all foreign contents return origin home as double counts deducted from the origin's value-added exports. However, at the same time, additive counts, *R*(+) return home from non-destinations or third parties which are part of foreign contents than the total foreign content:

$$\text{RETURN}(+) = \Sigma_{r,s,s\neq r}\boldsymbol{v}_r(\Sigma_{k\neq r,s}\boldsymbol{B}_{rk}\boldsymbol{E}_{ks}). \quad (24)$$

5. Numerical example

We provide a numerical example using an aggregated 3-country data shown by Table 2. 3-country consists of China (country 1), the USA (country 2) and the ROW (the rest of the world; country 3).

[Table 2 here]

Using the data of Table 2, the international Leontief inverse $\boldsymbol{B}= (\boldsymbol{B}rs)$ and value-added ratio vector $\boldsymbol{v}=(\boldsymbol{v}_r)$ are given by

$$\mathbf{A} = \begin{pmatrix} 0.5703 & 0.0036 & 0.0060 \\ 0.0059 & 0.4071 & 0.0131 \\ 0.0780 & 0.0416 & 0.4722 \end{pmatrix}, \ \boldsymbol{B} = \begin{pmatrix} 2.3323 & 0.0161 & 0.0268 \\ 0.0309 & 1.6897 & 0.0422 \\ 0.3471 & 0.1356 & 1.9019 \end{pmatrix},$$

$\boldsymbol{v} =$ (0.3458 0.5477 0.5088).

Destination-wise final demand is defined as

$$\boldsymbol{F}_1 = \begin{pmatrix} 1{,}968.1 \\ 11.9 \\ 110.5 \end{pmatrix}, \ \boldsymbol{F}_2 = \begin{pmatrix} 127.4 \\ 12{,}492.9 \\ 664.1 \end{pmatrix}, \ \boldsymbol{F}_3 = \begin{pmatrix} 267.3 \\ 352.4 \\ 29{,}423.9 \end{pmatrix} .$$

Using equations (2), we have

$$\begin{pmatrix} X_{12}^* \\ X_{22}^* \\ X_{32}^* \end{pmatrix} = \boldsymbol{BF}_2 = \begin{pmatrix} 515.7 \\ 21{,}141.6 \\ 3{,}001.3 \end{pmatrix}; \ \begin{pmatrix} X_{11}^* \\ X_{21}^* \\ X_{31}^* \end{pmatrix} = \boldsymbol{BF}_1 = \begin{pmatrix} 4{,}593.4 \\ 85.5 \\ 894.9 \end{pmatrix}; \ \begin{pmatrix} X_{13}^* \\ X_{23}^* \\ X_{33}^* \end{pmatrix} = \boldsymbol{BF}_3 = \begin{pmatrix} 1{,}418.4 \\ 1{,}845.2 \\ 56{,}101.1 \end{pmatrix}.$$

It follows from Definition 1 that

$v_1 X_{12}^* = 178.3$; $v_2 X_{21}^* = 46.8$; $v_1 X_{13}^* = 490.5$; $v_3 X_{31}^* = 455.3$; $v_2 X_{23}^* = 1{,}010.6$; $v_3 X_{32}^* = 1{,}527.0$.

On the other hand, from Table 2 and Definition 2 we have the following results of bilateral value-added exports:

$$\begin{pmatrix} X_{12}^{**} + X_{13}^{**} \\ -X_{21}^{**} \\ -X_{31}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} E_{12} + E_{13} \\ -E_{21} \\ -E_{31} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} 836.7 \\ -50.4 \\ -619.7 \end{pmatrix} = \begin{pmatrix} 1{,}934.1 \\ -85.5 \\ -894.9 \end{pmatrix};$$

$$\begin{pmatrix} -X_{12}^{**} \\ X_{21}^{**} + X_{23}^{**} \\ -X_{32}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -E_{12} \\ E_{21} + E_{23} \\ -E_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -210.6 \\ 1{,}187.0 \\ -1{,}624.3 \end{pmatrix} = \begin{pmatrix} -515.7 \\ 1{,}930.7 \\ -3001.3 \end{pmatrix};$$

$$\begin{pmatrix} -X_{13}^{**} \\ -X_{23}^{**} \\ X_{31}^{**} + X_{32}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -E_{13} \\ -E_{23} \\ E_{31} + E_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -626.1 \\ -1136.6 \\ 2{,}243.9 \end{pmatrix} = \begin{pmatrix} -1{,}418.4 \\ -1{,}845.2 \\ 3{,}896.2 \end{pmatrix}.$$

Clearly,

$$X_{12}^* = X_{12}^{**},\ X_{13}^* = X_{13}^{**},\ X_{21}^* = X_{21}^{**},\ X_{23}^* = X_{23}^{**},\ X_{31}^* = X_{31}^{**}, X_{32}^* = X_{32}^{**}.$$

Hence

$v_1 X_{12}^{*} = v_1 X_{12}^{**},\ v_1 X_{13}^{*} = v_1 X_{13}^{**}, v_2 X_{21}^{*} = v_2 X_{21}^{**}, v_2 X_{23}^{*} = v_2 X_{23}^{**},\ v_3 X_{31}^{*} = v_3 X_{31}^{**}, v_3 X_{32}^{*} = v_3 X_{32}^{**}$.

Theorem 1 is exemplified.

The results at the non-bilateral level are summarized by Tables 3 and 4.

Table 3. Decomposition of gross exports at non-bilateral level for 2005 in bln US$
Table 4. Decomposition of gross exports at non-bilateral level for 2005 in % of gross exports

[Table 3 and Table 4 here]

From Tables 3 and 4 we can witness the facts: the share of China's domestic content in its gross exports, 81% shows the smallest value of the three countries as its share and the amount of the foreign content show the largest values. At the non-bilateral level, the difference between the domestic content and the value-added exports is positive for all countries. This difference for China, 0.7% of its gross exports, is rather small because the double counts, which returns from the USA and the ROW to China, are very small. Particularly, the double count returning home from the USA, 0.03% of China's gross exports, appears to be negligible. The difference for the USA, 3.5% of its gross exports, is not so large due to the small double count returning home from China, 0.3% of USA's gross exports. The difference for the ROW show the largest share, 8.4% of its gross exports because both of double counts returning home from China and the USA are enough large. However, it is noteworthy that China's share of value-added exports, 80% is much smaller than USA's 89% and ROW's 88%.

[Table 5 and Table 6 here]

Tables 5 and 6 present our results at the bilateral level which illustrate Theorem 2. These tables show an interesting evidence that China's value-added exports to the USA is greater than China's domestic content induced by its gross exports to the US because the double count returning from the USA, 3.1% of share of gross exports, is smaller than the additive count returning from the third party non-destination ROW, 7.2%. The US value-added exports to China is also slightly larger than the US domestic content, despite the US large share of double count, 28.1% returning from China, because the US share of the additive count returning from the third party, non-destination ROW is also large as 28.4%, and greater than the double count returning from China. All other bilateral value-added exports are smaller than respective domestic content. In particular, ROW's share of value-added exports in its gross exports, 73.5%, is much less than its domestic content share, 97% due to the large double count returning from China. Table 5 also shows the identity between the total sum of double counts at the bilateral level (US$ billion 323) and that of double counts returning home (US$ billion 236.1) and no return part as double counts (US$ billion 86.9) at the non-bilateral level. As shown by Table 5, this no return part consists of return part as additive counts at the bilateral level.

6. Concluding remarks

We have presented a full description of an alternative accounting system of gross exports at both the non-bilateral level and bilateral level in place of that of Koopman et al. (2014) by using our definition of value-added exports. We demonstrated that we could trace value added and double accounting in gross exports across countries in a straightforward manner when we employ our representation of value-added exports (Definition 2). As was shown, tracing value-added and double counting in gross exports became dramatically more difficult and complicated when moving from a two-country to three-country or multi-country world. We demonstrated that the value of bilateral value-added exports can be greater than the respective domestic content due to the additive count returning from the third parties, which can be larger than the double count returning from the destination country. Koopman et al. (2014) rightly provided the decomposition into the domestic and foreign content of gross exports in the first stage. However, they failed to present an easily understandable exposition of further decomposition into value-added exports, terms returning home as double or additive counts at both non-bilateral and bilateral levels. This is simply due to their employment of Johnson and Noguera's definition of value-added trade, where double accounting terms are not explicitly or visually shown. In contrast, our definition (Definition 2) straightforwardly shows logic and elements of decompositions. We also provided a numerical and empirical example for a three-country case (China, the USA, and the ROW), using aggregated World Input-Output Data. Needless to say, we need further theoretical empirical investigation.

Appendix

1. Traditional value-added trade on the country base

Let us consider a 2-country case with many sectors where country 1 is an arbitrary country, such as the USA, China, or Japan, and country 2 is the rest of the world (ROW).

From equation (4), the local equilibrium condition for country 1 is given by

$$\boldsymbol{X}_1^* = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-\mathbf{1}}\widetilde{\boldsymbol{Y}}_1, = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-\mathbf{1}}(\boldsymbol{E}_{12}+\boldsymbol{Y}_{11}); \qquad (4')$$

$$\boldsymbol{X}_1^* = \boldsymbol{B}^1\widetilde{\boldsymbol{Y}}_1 = \boldsymbol{B}^1(\boldsymbol{E}_{12}+\boldsymbol{Y}_{11}), \text{where } \boldsymbol{B}^1 = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-\mathbf{1}}.$$

$\boldsymbol{E}_{12} = \boldsymbol{E}_1$ is a column vector of country 1's gross exports to the world. $\boldsymbol{Y}_{11}$ is a column vector of country 1's final demand for its own domestic products. Generally, $\boldsymbol{B}^1 \neq \boldsymbol{B}_{11}$, while the global equilibrium and local equilibria yield the same solution in the international input–output system.

Country 1's traditional domestic contents of value-added, induced by its exports to country 2 or the world, are defined as

$$\underline{DV_1} = \boldsymbol{v}_1(\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12}. \qquad (A1)$$

This definition, given by Miyazawa (1975), has widely been utilized in Japan and included in the Japanese Statistics Bureau's official reports (the latest version can be seen on the website).[7]

Country 1's exports need its imports from country 2, which amount to

$$FMX_1 = \boldsymbol{u}_n\boldsymbol{A}_{21}(\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12}. \qquad (A2)$$

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[7] An Excel file of "Gross Value Added Induced by Individual Final demand Items" http://www.soumu.go.jp/english/dgpp_ss/data/io/io11.htm (Access on July 1, 2016).

In view of our price equation, $\boldsymbol{v}_1 + \boldsymbol{u}_n\boldsymbol{A}_{21} = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})$, we have

$$\boldsymbol{v}_1(\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12} + \boldsymbol{u}_n\boldsymbol{A}_{21}(\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12}.$$

$$= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})(\mathbf{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12} = \boldsymbol{u}_n\boldsymbol{E}_{12};$$

$$E_1 = \underline{DV_1} + FMX_1. \tag{A3}$$

Country 1's total traditional value-added exports to the world, $\underline{DV_1}$, *plus* imports induced by exports, FMX_1, should equal its total gross exports to the world, E_1 (Miyazawa, 1975). This identity implies that gross exports can be decomposed into value added and imports induced by gross exports on the country base. This is a corollary of identity between the GDP on the production side and the GDP on the expenditure side. In the definition of the traditional value-added exports on the country base, the exports of intermediates are exogenously given. When we measure traditional value-added trade on the country base, we do not have to be concerned with endogeneity issues of the ROW. Given the information of a country's non-competitive input-output table and gross exports by country and sector, we can easily compute the country's traditional domestic value-added. However, this approach is insufficient for tracing international transfers of value added and outputs in the world.

Imports of country 1, which are exports of country 2, generated by its exports also, in turn, induce value added in country 2, that is to say, foreign value added on the country base, $\underline{FV}_1$. When we apply the country base equation to country 2, we have

$$\underline{FV_1} = \boldsymbol{v}_2(\mathbf{I}_n - \boldsymbol{A}_{22})^{-1}\boldsymbol{A}_{21}(\mathbf{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12}.$$

If $\boldsymbol{v}_2(\mathbf{I}_n - \boldsymbol{A}_{22})^{-1} = \boldsymbol{u}_n$, we can write equation (A1) as

$$E_1 = \underline{DV_1} + \underline{FV_1}.$$

If and only if $\boldsymbol{A}_{12} = \boldsymbol{O}$, generally we can obtain $\boldsymbol{v}_2(\mathbf{I}_n - \boldsymbol{A}_{22})^{-1} = \boldsymbol{u}_n$. This might be a case discussed in Los et al. (2016). However, in general, $\boldsymbol{A}_{12} \neq \boldsymbol{O}$. Therefore, when we employ separately each country's traditional domestic value added, generally

$$E_1 \neq \underline{DV_1} + \underline{FV_1}.$$

This implies that, in a separate use of traditional domestic value-added, gross exports cannot be absorbed fully or decomposed into value-added. Only when we use a global Leontief inverse, we reach the basic identity equation (12) in the text where all gross exports of a country are fully absorbed and decomposed into domestic and foreign value-added. However, it is noteworthy to mention a possible joint use of traditional and new concepts of domestic value-added, as Koopman et al. (2014) attempted. By exploring the domestic value-added and induced imports on the country base into equation (12) on the international base, for instance, we have the following additional identity:

$$E_1 = \underline{DV_{\mathbf{1}}} + (DV_{\mathbf{1}} - \underline{DV_{\mathbf{1}}}) + FMX_1 + (FV_1 - FMX_1). \qquad \text{(A4)}$$

Equation (A4) means that total exports (LHS) = (term 1 of RHS), the domestic value-added on the country base + (term 2 of RHS), the difference between the domestic value-added on the country base and that on the international base + (term 3 of RHS), induced imports on the country base + (term 4 of RHS), the difference between foreign value-added on the international base and induced imports on the country base. Obviously, the smaller term 2 and term 4 of RHS become, the more we can rely upon results on the country base. Empirically, terms 2 and 4 are rather negligible in comparison with results on the country base because each country's input coefficients of domestic intermediates are much larger than those of imported/exported intermediates.

If we have complete data on international input-output tables (Table 1), in principle, we do not have to be concerned with equation (A4) because this equation is not essential for the decomposition of gross exports on the international base. As can be seen, Koopman et al. (2014) shifted the main focus from $DV_{\mathbf{1}}$ (in Koopman et al. 2010) to $\underline{DV_{\mathbf{1}}}$ without any explanation. This shift makes their decomposition a hard read, while this would need further research.

2. The double use of intermediate imports/exports on the international base

On the international base, local domestic value added can be written as

$$\underline{DV_{\mathbf{1}}} = \boldsymbol{v}_1(\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}\boldsymbol{E}_{12} = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{A}_{12}\boldsymbol{X}^*{}_2 + \boldsymbol{Y}_{12}). \qquad \text{(A1')}$$

The term of the export of intermediates, $\boldsymbol{A}_{12}\boldsymbol{X}^*{}_2$, is considered an endogenous variable in the international input-output system. Throughout this paper, on the international base, we also employ $\boldsymbol{v}_r\boldsymbol{B}_{rs}\boldsymbol{E}_{rs}$ or $\boldsymbol{B}_{rs}\boldsymbol{E}_{rs}$. $\boldsymbol{B}_{rs}$ embodies $\boldsymbol{A}_{rs}$, while $\boldsymbol{E}_{rs}$ includes the term of exports/imports of intermediates, $\boldsymbol{A}_{rs}\boldsymbol{X}^*{}_s$. This implies that we doubly use endogenous parts of the international input-output system as endogenous and exogenous variables. Therefore, some input-output experts (here called *fundamentalists*), including Los et al. (2016), may think this operation ($\boldsymbol{B}_{rs}$ post-multiplied by $\boldsymbol{E}_{rs}$, including exports of intermediates) is not right because they believe that the Leontief inverse, irrespective of national or international frameworks, should be post-multiplied by exogenous final demand terms. For fundamentalists, only Johnson and Noguera (2012) can be accepted because their paper provides the definition of value-added exports by using $\boldsymbol{B}_{rs}$ post-multiplied by the final demand, $\boldsymbol{F}_s$, excluding exports of intermediates. For fundamentalists, all other papers, including those of Trefler and Zhu (2010), Koopman et al. (2010; 2014), and this paper, are not plausible from an economics point of view. Their idea may be true for the use of Leontief equations for projections. However, *given input-output structures*, all parts of endogenous intermediate transactions are also given. To analyze the given input-output structure, our double use in the system *with fixed and constant input coefficients and value-added coefficients* can be allowed, as mathematics and Trefler and Zhu (2010) show the feasibility and fruitfulness of our operations.

Let us consider the most orthodox Leontief system with competitive imports on the country base for country 1:

$$\boldsymbol{X}_1 = \widetilde{\boldsymbol{A}}_{11}\boldsymbol{X}_1 + \boldsymbol{Y}_{11} + \boldsymbol{E}_1 - \boldsymbol{M}_1;\ \boldsymbol{X}_1 = \left(\boldsymbol{I}_n - \widetilde{\boldsymbol{A}}_{11}\right)^{-1}(\boldsymbol{Y}_{11} + \boldsymbol{E}_1 - \boldsymbol{M}_1),$$

where $\widetilde{\boldsymbol{A}}_{11}$ is a matrix of input coefficients of intermediates, including domestic and imported intermediates; $\boldsymbol{M}_1$ is a vector of all imports for intermediate demand and final demand as well. Can input-output fundamentalists allow $\left(\boldsymbol{I}_n - \widetilde{\boldsymbol{A}}_{11}\right)^{-1}$ post-multiplied by $\boldsymbol{M}_1$ despite the double use of imports of intermediates in $\widetilde{\boldsymbol{A}}_{11}$, $\widetilde{\boldsymbol{A}}_{11}\boldsymbol{X}_1$, and $\boldsymbol{M}_1$?[8] If the answer is yes, our double use should also be accepted.

It is noteworthy to learn that, through decomposition processes, all the double uses of intermediate exports or imports are eliminated, which is shown by the identity between the total amount of a country's value-added trade balances with other countries

[8] Input-output economists recognize that the exogenous treatment of imports should be improved because imports largely depend on domestic production activities. Along this line, there have been several attempts in the input-output literature to make imports endogenous, including this paper. However, this is the story of the second step in input-output analysis. The key problem of input-output analysis, as posed by many economists, is its exogenous treatment of all terms of final demand.

and that of its gross trade balances or, in the conventional terminology, net exports (Kuboniwa, 2014a, 2015). Regardless, you should not stop or remain at the entrance of decompositions. Go further along with decompositions shown in the text, then you would arrive at a full understanding of movements of the value-added flows, induced by the gross trade, across the world.

Acknowledgements

The author thanks Dr. Jiemin Guo, the U.S. Bureau of Economic Analysis, for his helpful comments on an earlier version of this paper with encouragement. He also thanks Dr. Yoshisada Shida, Hitotsubashi University, for his diligent cooperation. He is grateful for the JSPS scientific support: Grant-in-Aid (A) numbers 15H01959 and (B) 24330085.

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Table 1. Data structure of an international input-output table

	Intermediate demand/input								Final demand (destination)								
	Country 1	Country 2	…	Country r	…	Country s	…	ROW R	Country 1	Country 2	…	Country r	…	Country s	…	ROW R	Output
									$\boldsymbol{F}_1$	$\boldsymbol{F}_2$	…	$\boldsymbol{F}_r$	…	$\boldsymbol{F}_s$	…	$\boldsymbol{F}_R$	$\boldsymbol{X}$
Country 1	$\underline{\boldsymbol{X}}_{11}$	$\underline{\boldsymbol{E}}_{12}$	…	$\underline{\boldsymbol{E}}_{1r}$	…	$\underline{\boldsymbol{E}}_{1s}$	…	$\underline{\boldsymbol{E}}_{1R}$	$\boldsymbol{Y}_{11}$	$\boldsymbol{Y}_{12}$	…	$\boldsymbol{Y}_{1r}$	…	$\boldsymbol{Y}_{1s}$	…	$\boldsymbol{Y}_{1R}$	$\boldsymbol{X}_1$
Country 2	$\underline{\boldsymbol{E}}_{21}$	$\underline{\boldsymbol{X}}_{22}$	…	$\underline{\boldsymbol{E}}_{2r}$	…	$\underline{\boldsymbol{E}}_{2s}$	…	$\underline{\boldsymbol{E}}_{2R}$	$\boldsymbol{Y}_{21}$	$\boldsymbol{Y}_{22}$	…	$\boldsymbol{Y}_{2r}$	…	$\boldsymbol{Y}_{2s}$	…	$\boldsymbol{Y}_{2R}$	$\boldsymbol{X}_2$
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Country r	$\underline{\boldsymbol{E}}_{r1}$	$\underline{\boldsymbol{E}}_{r2}$	…	$\underline{\boldsymbol{X}}_{rr}$	…	$\underline{\boldsymbol{E}}_{rs}$	…	$\underline{\boldsymbol{E}}_{rR}$	$\boldsymbol{Y}_{r1}$	$\boldsymbol{Y}_{r2}$	…	$\boldsymbol{Y}_{rr}$	…	$\boldsymbol{Y}_{rs}$	…	$\boldsymbol{Y}_{rR}$	$\boldsymbol{X}_r$
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Country s	$\underline{\boldsymbol{E}}_{s1}$	$\underline{\boldsymbol{E}}_{s2}$	…	$\underline{\boldsymbol{E}}_{sr}$	…	$\underline{\boldsymbol{X}}_{ss}$	…	$\underline{\boldsymbol{E}}_{sR}$	$\boldsymbol{Y}_{s1}$	$\boldsymbol{Y}_{s2}$	…	$\boldsymbol{Y}_{sr}$	…	$\boldsymbol{Y}_{ss}$	…	$\boldsymbol{Y}_{sR}$	$\boldsymbol{X}_s$
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
ROW R	$\underline{\boldsymbol{E}}_{R1}$	$\underline{\boldsymbol{E}}_{R2}$	…	$\underline{\boldsymbol{E}}_{Rr}$	…	$\underline{\boldsymbol{X}}_{Rs}$	…	$\underline{\boldsymbol{X}}_{RR}$	$\boldsymbol{Y}_{R1}$	$\boldsymbol{Y}_{R2}$	…	$\boldsymbol{Y}_{Rr}$	…	$\boldsymbol{Y}_{Rs}$	…	$\boldsymbol{Y}_{RR}$	$\boldsymbol{X}_R$
Value-added	$\boldsymbol{V}_1$	$\boldsymbol{V}_2$	…	$\boldsymbol{V}_r$	…	$\boldsymbol{V}_s$	…	$\boldsymbol{V}_R$									
Output	$\boldsymbol{X}_1$	$\boldsymbol{X}_2$	…	$\boldsymbol{X}_r$	…	$\boldsymbol{X}_s$	…	$\boldsymbol{X}_R$									

Notes:

$\underline{\boldsymbol{E}}_{rs}$ $(s \neq r)$: country r's gross export matrix of intermediate goods to country s or country s's gross import matrix of intermediate goods from country r.

$\underline{\boldsymbol{X}}_{rr}$: country r's input matrix of domestically produced intermediate goods.

$\boldsymbol{Y}_{rs}$ $(s \neq r)$: country r's gross export matrix of final goods to country s or country s's gross import matrix of final goods from country r.

$\boldsymbol{Y}_{rr}$: country r's input matrix of domestically produced final goods.

$\boldsymbol{V}_r$: country r's value-added vector.

Table 2. Aggregated 3-country world input-output table (China-USA-ROW): 2005

(in producer prices; current US$ bln)

	Intermediate demand			Intermediate demand	Final demand			Final demand	Output
	country 1 China	country 2 USA	country 3 ROW		country 1 China	country 2 USA	country 3 ROW		
country 1 China	3,722.7	83.2	358.8	4,164.6	1,968.1	127.4	267.3	2,362.9	6,527.5
country 2 USA	38.5	9,392.4	784.2	10,215.1	11.9	12,492.9	352.4	12,857.2	23,072.3
country 3 ROW	509.1	960.2	28,329.4	29,798.7	110.5	664.1	29,423.9	30,198.5	59,997.3
Intermediate input	4,270.3	10,435.7	29,472.4	44,178.4	2,090.5	13,284.4	30,043.7	45,418.6	89,597.0
Value-added	2,257.2	12,636.6	30,524.9	45,418.6					
Output	6,527.5	23,072.3	59,997.3	89,597.0					

Sources: Author's calculations using WIOD.

Notes:

1. Producer prices = basic prices + net taxes on products.

2. For China and USA, WIOD employs data in producer prices, while, for EU countries and some other countries, included in ROW, WIOD uses data in basic prices. We converted data for ROW into data in producer prices.

Table 3. Decomposition of gross exports at non-bilateral level for 2005 in bln US$

Origin r	Destination s (s', s'')	Gross exports E_r	Domestic content DV_r	double count (-)	return home	from country s'	return home	from country s''	Value-added exports E_r^{va}	Foreign content FV_r	return home as double count	return	to country s'	return	to country s''	no return as double count
1 China	2,3	836.7	674.8	6.0	0.3	from 2	5.7	from 3	668.8	161.9	114.1	3.6	to 2	110.6	to 3	47.8
2 USA	1,3	1,187.0	1,098.5	41.1	3.6	from 1	37.5	from 3	1,057.4	88.5	78.7	0.3	to 1	78.4	to 3	9.8
3 ROW	1,2	2,243.9	2,171.3	189.0	110.6	from 1	78.4	from 2	1,982.3	72.7	43.3	5.7	to 1	37.5	to 2	29.4

Table 4. Decomposition of gross exports at non-bilateral level for 2005 in % of gross exports (% of gross exports)

Origin r	Destination s (s', s'')	Gross exports E_r	Domestic content DV_r	double count (-)	return home	from country s'	return home	from country s''	Value-added exports E_r^{va}	Foreign content FV_r	return home as double count	return	to country s'	return	to country s''	no return as double count
1 China	2,3	100.0	80.7	0.72	0.03	from 2	0.69	from 3	79.9	19.3	13.6	0.4	to 2	13.2	to 3	5.7
2 USA	1,3	100.0	92.5	3.5	0.3	from 1	3.2	from 3	89.1	7.45	6.63	0.02	to 1	6.61	to 3	0.82
3 ROW	1,2	100.0	96.8	8.4	4.9	from 1	3.5	from 2	88.3	3.2	1.9	0.3	to 1	1.7	to 2	1.3

Table 5. Decomposition of gross exports at bilateral level for 2005 in bln US$.

Origin r	Destination s	Gross exports E_{rs}	Domestic content DV_{rs}	Double count (-); return	from country s	Additive count (+); return	from country k ($k \neq r,s$)	Value-added exports E_{rs}^{va}	Foreign content FV_{rs}	Return to country s as double count (-)		Return to country k as addditive count (+)	
1 China	2	210.6	169.8	6.6	from 2	15.1	from 3	178.3	40.7	3.6	to 2(-)	37.2	to 3(+)
	3	626.1	505.0	20.8	from 3	6.3	from 2	490.5	121.2	110.6	to 3(-)	10.6	to 2(+)
2 USA	1	50.4	46.6	14.1	from 1	14.3	from 3	46.8	3.8	0.3	to 1(-)	3.5	to 3(+)
	3	1,136.6	1,051.9	51.9	from 3	10.6	from 1	1,010.6	84.7	78.4	to 3(-)	6.3	to 1(+)
3 ROW	1	619.7	599.6	147.8	from 1	3.5	from 2	455.3	20.1	5.7	to 1(-)	14.3	to 2(+)
	2	1,624.3	1,571.7	81.9	from 2	37.2	from 1	1,527.0	52.6	37.5	to 2(-)	15.1	to 1(+)

Table 6. Decomposition of gross exports at bilateral level for 2005 in % of gross exports

Origin r	Destination s	Gross exports E_{rs}	Domestic content DV_{rs}	Double count (-); return	from country s	Additive count (+); return	from country k ($k \neq r,s$)	Value-added exports E_{rs}^{va}	Foreign content FV_{rs}	Return to country s as double count (-)		Return to country k as addditive count (+)	
1 China	2	100.0	80.7	3.1	from 2	7.2	from 3	84.7	19.3	1.7	to 2(-)	17.7	to 3(+)
	3	100.0	80.7	3.3	from 3	1.0	from 2	78.3	19.3	17.7	to 3(-)	1.7	to 2(+)
2 USA	1	100.0	92.5	28.1	from 1	28.4	from 3	92.9	7.5	0.6	to 1(-)	6.9	to 3(+)
	3	100.0	92.5	4.6	from 3	0.9	from 1	88.9	7.5	6.9	to 3(-)	0.6	to 1(+)
3 ROW	1	100.0	96.8	23.8	from 1	0.6	from 2	73.5	3.2	0.9	to 1(-)	2.3	to 2(+)
	2	100.0	96.8	5.0	from 2	2.3	from 1	94.0	3.2	2.3	to 2(-)	0.9	to 1(+)

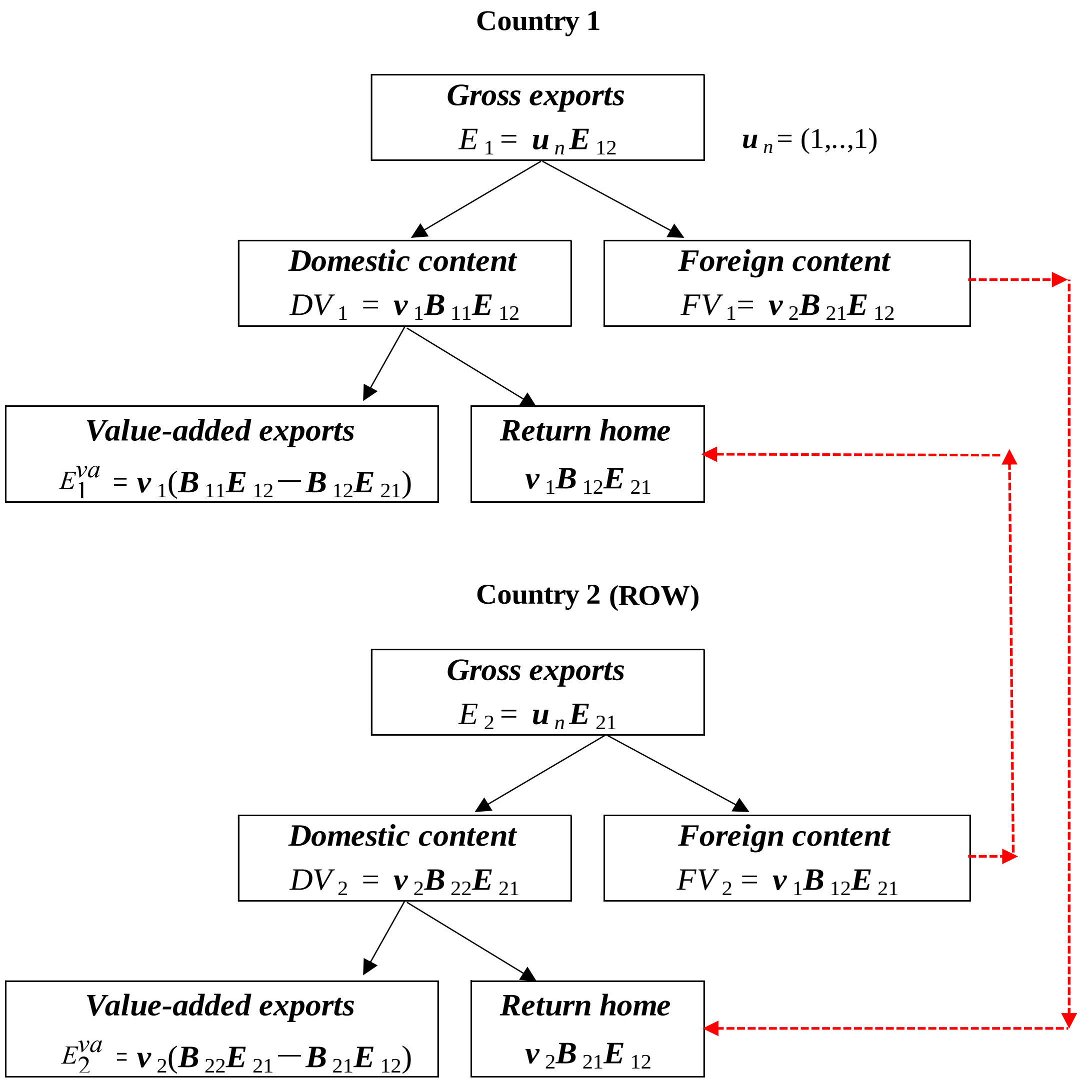
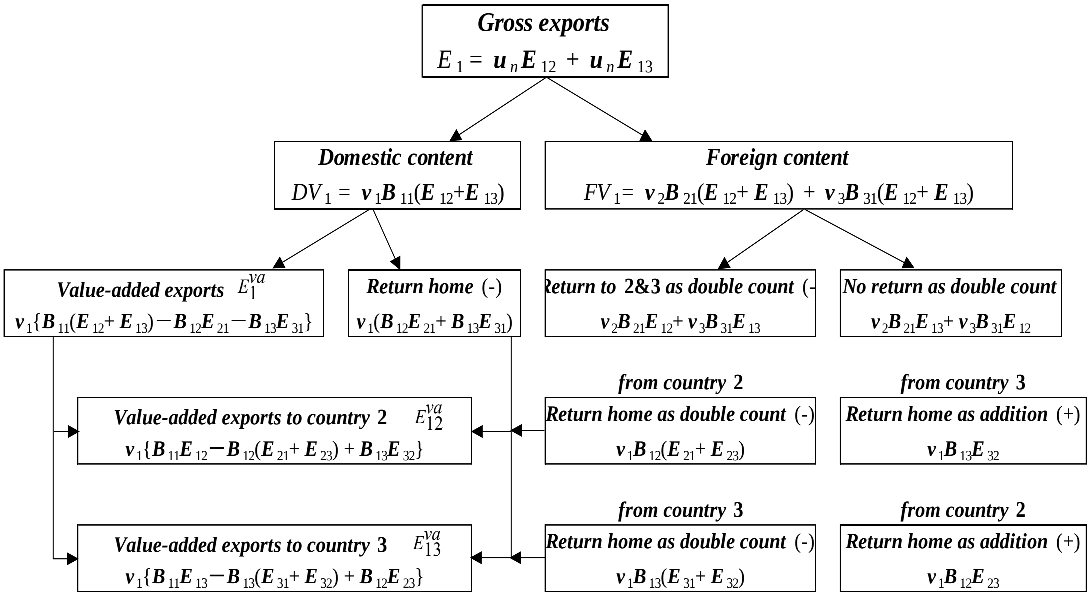


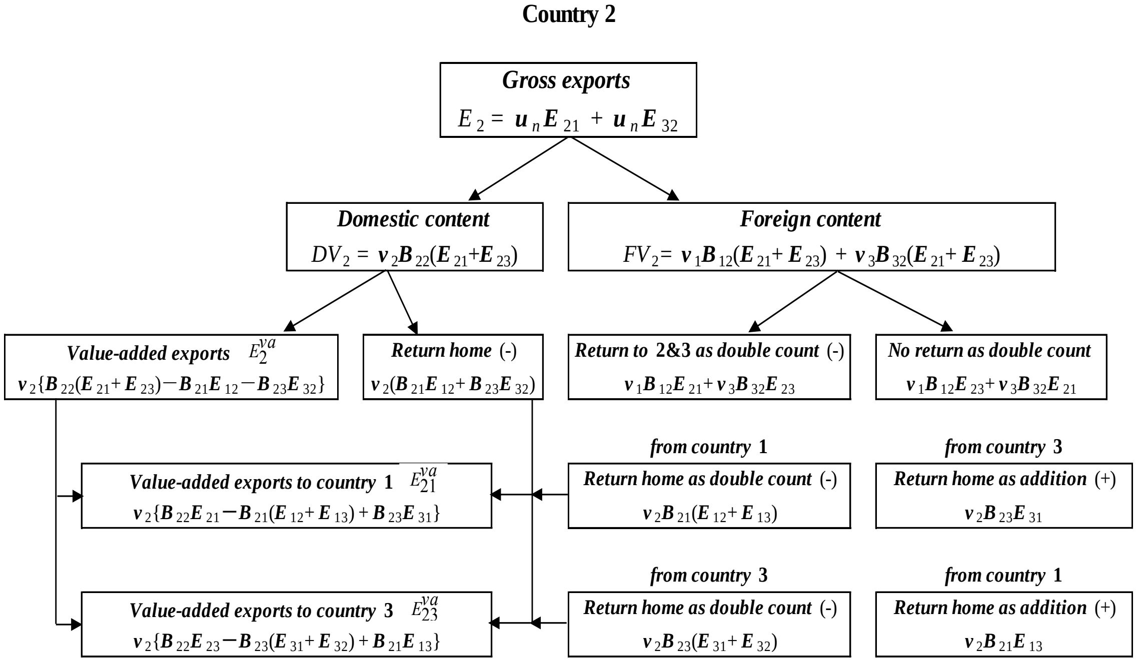
Figure 1. Decomposition of gross exports and double counts: a two-country case with many sectors

(Panel 1 of Figure 2)

Country 1



(Panel 2 of Figure 2)



(Panel 3 of Figure 2)

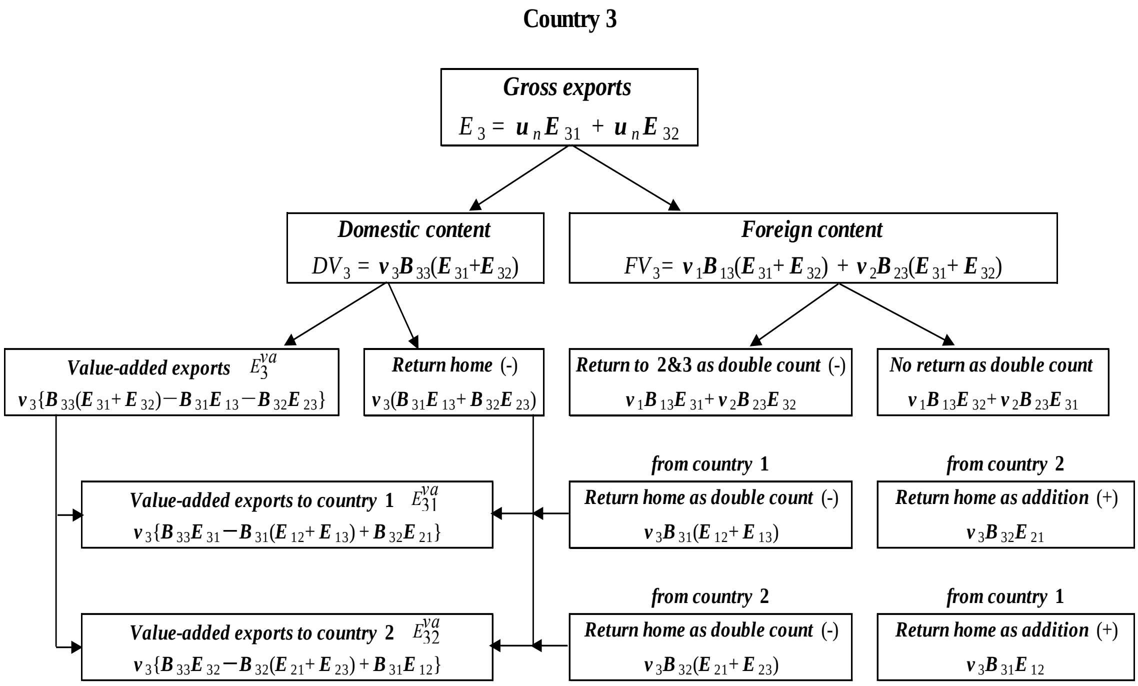


Figure 2. Decomposition of gross exports, double counts and additive counts: a 3-country case with many sectors